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ON THE BRITTLE-DUCTILE TRANSITION PRESSURE

by

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SUMMARY

A theory for the change from brittle to ductile fracture behaviour with increasing superimposed hydrostatic pressure is proposed, and is extended to account for the effect of temperature on the transition. The theory agrees well with empirical relationships developed previously by other workers, and shows that an effect due to grain size is to be expected.

C O N T E N T S

Distribution Groups C, J

NOTATION

$A, A', B, B', k, k_0,$ $\alpha, \beta, \beta', \epsilon, \sigma_{00}, \chi$	Constants
b	Burger's vector
d	Mean grain diameter
G	Rigidity modulus
l	Length of crack
n and n'	Number of dislocations
p	Hydrostatic pressure
p_c	Brittle-ductile transition pressure
r	Effective radius of stress field around a dislocation crack
T	Temperature
T_c	Brittle-ductile transition temperature
W	Total energy of crack
γ'	Effective surface energy of crack
ζ	$= 16\ G\gamma'/M(1 - \nu)\ k_0$
θ	Angle between crack and slip plane
λ	Constant $= \alpha k_0$
ν	Poisson's ratio
ξ	Constant $= \alpha\sigma_{00}$
σ	Applied tensile stress
σ_f	Fracture stress
σ_0	Frictional stress opposing the motion of a free dislocation

1. INTRODUCTION

It has been reported by Pugh[1] that some metals, notably zinc and bismuth, which are normally brittle in tension at NTP, become markedly ductile when tested under hydrostatic pressure. This transition from brittle to ductile behaviour is very sharp, the specimens being either very brittle or very ductile. Galli and Gibb[2] have suggested the empirical relationship between the brittle to ductile transition temperature, T_c, and hydrostatic pressure, p, for molybdenum

$$T_c = \frac{A}{\ln(\sigma_f + p) + B}, \tag{1}$$

where A and B are constants and σ_f is the fracture stress at atmospheric pressure. Pugh[3] suggested for zinc the empirical relationship

$$T_c = A' - B'p, \tag{2}$$

where A' and B' are constants. A theory for the transition is given below.

2. THEORY OF THE TRANSITION PRESSURE

Near the brittle to ductile transition, brittle fracture occurs when the applied stress is sufficiently high to propagate a dislocation crack in a Griffith manner. If the applied stress is not high enough for this, fracture may occur by the linkage of many small cracks. It is thought that the propagation of a crack requires more energy than its initiation[4,5], and the analysis is based on this criterion. The calculation is similar to one due to Petch for the brittle-ductile transition temperature in mild steel[6].

Consider a crack of length l at an angle θ to the slip plane, wedged open by n dislocations of Burger's vector, b, under the action of the applied tensile stress, σ, at an angle $\pi/4$ to the slip plane, Fig. 1 (the most favoured slip lines are those at $\pi/4$ to the tensile stress). Let the applied hydrostatic pressure be p. If we assume that the hydrostatic pressure affects only the work done in opening the crack, then the energy, W, of the crack under this stress system, represented in Fig. 1, is composed of the following.

(1) The elastic energy of the stress field set up by the crack; this has been calculated by Stroh[7] to be $[(n^2b^2G)/\{4\pi(1-\nu)\}] \ln 4r/l$, where G is the rigidity modulus, ν the Poisson's ratio and r the effective radius of the stress field.

(2) The surface energy of the crack; this is given by $2l\gamma'$, where γ' is the effective surface energy of the crack. This effective surface energy term includes the plastic work associated with the growth of the crack, and may be much greater than the true surface energy.

(3) The elastic energy of the crack in the applied stress field; Stroh[8] calculates this to be

$$-\frac{\pi(1-\nu)\sigma^2 l^2}{8G}.$$

The work of Sack[9] suggests that no great error results in ignoring the effect of the hydrostatic pressure.

(4) The energy due to the increase in volume on opening the crack; this

consists of two parts, namely (a) the work done by the component of tensile stress normal to the crack, $(-nbl/2)\sigma \sin\{\theta - (\pi/4)\}$, and (b) the work done against the hydrostatic pressure $(nbl/2)p$.

The total energy of the crack is thus

$$W = \frac{n^2b^2G}{4\pi(1-\nu)} \ln\frac{4r}{l} + 2l\gamma' - \frac{\pi(1-\nu)\sigma^2l^2}{8G} - \frac{nbl}{2}\sigma \sin\left(\theta - \frac{\pi}{4}\right) + \frac{nbl}{2}p. \tag{3}$$

Strictly, this equation applies only to a two-dimensional model but Sack[9], who extended the argument to a penny-shaped crack, showed that the difference will be a numerical factor only.

For the crack to spread under the applied stress, W must decrease as l increases, and the length of the crack at equilibrium will be given by $dW/dl = 0$. Thus

$$\frac{dW}{dl} = 0 = -\frac{n^2b^2G}{4\pi(1-\nu)}\frac{1}{l} + \frac{nb}{2}\left\{\frac{4\gamma'}{nb} + p - \sigma\sin\left(\theta - \frac{\pi}{4}\right)\right\} - \frac{\pi c}{4G}(1-\nu)\sigma^2. \tag{4}$$

Rearranging equation (4) we get

$$-\frac{\pi}{4G}(1-\nu)\sigma^2l^2 - \frac{nb}{2}\left\{\frac{4\gamma'}{nb} + p - \sigma\sin\left(\theta - \frac{\pi}{4}\right)\right\} l + \frac{n^2b^2G}{4\pi(1-\nu)} = 0. \tag{5}$$

The critical length of crack will occur when the roots of equation (5) are equal, and this will happen when

$$\frac{4\gamma'}{nb} + p_c - \sigma\sin\left(\theta - \frac{\pi}{4}\right) = \sigma, \tag{6}$$

where p_c is now the critical hydrostatic pressure for the transition from brittle to ductile fracture. Rearranging equation (6) we get as the critical condition

$$nb\left\{\sigma + \sigma\sin\left(\theta - \frac{\pi}{4}\right) - p_c\right\} = 4\gamma'. \tag{7}$$

If the left-hand side of equation (7) exceeds $4\gamma'$, then the crack will spread catastrophically. In practice, especially with the more ductile metals, plastic blunting of the crack may occur, in which case a higher value of σ would be required for a given p.

The value of n is obtained from the work of Eshelby, Frank and Nabarro[10]. If a dislocation source is activated near the centre of a grain of diameter d, the maximum length of slip plane on which a crack can form is $d/2$. The applied shear stress will be $\frac{1}{2}(\sigma - \sigma_0)$, where $\frac{1}{2}\sigma_0$ is the frictional shear stress opposing the motion of a free dislocation. The number of dislocations, n', that can be packed into a length $d/2$ under this applied stress is given by

$$\frac{1}{2}(\sigma - \sigma_0) = \frac{2Gn'b}{\pi(1-\nu)d}$$

or

$$n' = \frac{\pi}{4Gb}(1-\nu)(\sigma - \sigma_0)d. \tag{8}$$

Now Stroh[8] has shown that the most difficult step in the coalescence of dislocations is the coalescence of the first two, the stress to add subsequent dislocations falling progressibely. Therefore very few dislocations will remain on the slip plane, the majority being in the dislocation crack. We may thus write

$$n = n'.$$

By substituting the right-hand side of equation (8) for n in equation (7), we get

$$\frac{\pi}{4G} (1 - \nu)(\sigma - \sigma_0)d\left\{\sigma + \sigma \sin \left(\theta - \frac{\pi}{4}\right) - p_c\right\} = 4\gamma'. \tag{9}$$

Now consider the value of σ which will be the flow stress at which the brittle fracture occurs. Armstrong and others (11) have shown that the flow stresses, σ_f, at equal strains of many metals and alloys obey the Petch relationship

$$\sigma_f = \sigma_0 + kd^{-\frac{1}{2}}, \tag{10}$$

where k is a constant for a particular metal and strain. The amount of strain occurring prior to brittle fracture is generally small, and any variations due to grain size are likely to be of secondary importance. We may thus put

$$\sigma = \sigma_0 + kd^{-\frac{1}{2}}, \tag{11}$$

since σ will be the flow stress at fracture.

Since $(\theta - \pi/4)$ is positive the value of $\sigma \sin (\theta - \pi/4)$ will have a maximum value of σ and a minimum value of 0, so we may write

$$\sigma + \sigma \sin \left(\theta - \frac{\pi}{4}\right) = \alpha\sigma, \tag{12}$$

where $1 \leqslant \alpha \leqslant 2$. Substituting appropriate terms from equations (11) and (12) in equation (9) gives

$$\frac{\pi}{4G} (1 - \nu)kd^{\frac{1}{2}} (\alpha\sigma - p_c) = 4\gamma'.$$

Rearranging, we get

$$p_c = \alpha\sigma - \left\{\frac{16G\gamma'}{\pi(1 - \nu)k}\right\} d^{-\frac{1}{2}} \tag{13a}$$

or

$$p_c = \alpha\sigma_0 - \left\{\frac{16G\gamma'}{\pi(1 - \nu)k} - \alpha k\right\} d^{-\frac{1}{2}}. \tag{13b}$$

Now the terms in the brackets are substantially constant, for constant temperature, so we may write

$$p_c = \alpha\sigma - \beta d^{-\frac{1}{2}} \tag{14a}$$

or

$$p_c = \alpha\sigma_0 - \beta' d^{-\frac{1}{2}}. \tag{14b}$$

If the hydrostatic pressure, p, exceeds the critical value, p_c, then ductile fracture will occur. Conversely, brittle fracture will occur if p is less than p_c.

The constants cannot be accurately determined theoretically in view of the assumption that a two-dimensional analysis is applicable to a three-dimensional problem. However, it is expected that values of α, β and β' derived from known values of the various parameters will be of the right order of magnitude.

3. THE BRITTLE-DUCTILE TRANSITION PRESSURE IN ZINC

The work of Pugh[1,3] shows that brittle fracture in zinc just below the transition pressure occurs after approximately 5 per cent strain and that the transition pressure for the particular grain size used ($d \simeq \frac{1}{2}$ mm) was 7.5 kg/mm^2. The results of Armstrong and others[11] show that for 5 per cent strain k is 0.76 kg/mm$^{\frac{3}{2}}$ and σ_0 is 5.8 kg/mm^2. We may thus write from equation (14b)

$$7.5 = 5.8\alpha - 1.4\beta'. \tag{15}$$

From Armstrong and others[11] the transition at atmospheric pressure would appear to be at a $d^{-\frac{1}{2}}$ value of approximately 2.8 mm$^{-\frac{1}{2}}$. From this information we get

$$0 = 5.8\alpha - 2.8\beta'. \tag{16}$$

At this point it should be noted that zinc slips only on (0001), so that $\theta = 0$ and in our two-dimensional model α should be approximately 1.7. However, the value of α will be modified by the three-dimensional nature of the problem, and Sack's analysis[9] suggests a value which will be greater by a factor of approximately 1.6, i.e. 2.7. Zinc also twins very readily, and it is known that cleavage fractures can originate at twin intersections. This will also modify the value of α. The work of Armstrong and others[11] suggests an increase in k and a decrease in σ_0 when twin boundaries are counted as well as grain boundaries. A simple calculation shows that the slip systems in zinc are unlikely to be affected by the applied hydrostatic pressure.

From equations (15) and (16) we get

$$\alpha = 2.59$$

$$\beta' = 5.36 \text{ kg/mm}^{-\frac{3}{2}}.$$

The value of γ' corresponding to β' is 1780 ergs/cm^2. This is about double the true surface energy of zinc and is lower than expected. The value of α is also lower ~~higher~~ than expected. However, the approximate data used would probably account for this.

4. EFFECT OF TEMPERATURE ON THE BRITTLE-DUCTILE TRANSITION PRESSURE

In equation (13) the main temperature-dependent parameters are k, σ_0 and γ'. The parameter k, which is a measure of the dislocation locking strength and is thus subject to thermal activation, can be roughly approximated to $k_0 e^{-\epsilon T}$, where k_0 and ϵ are constants. The variation of σ_0 with temperature, at least in the case of b.c.c. metals, stems from a large Peierls-Nabarro stress. In the case of h.c.p. metals, the Peierls-Nabarro stress is small and so the temperature variation of σ_0 can be ignored. Heslop and Petch[12] have shown that the Peierls-Nabarro stress can be represented by $\sigma_{00} e^{-\chi T}$, where σ_{00} and χ are constants. The whole of σ_0 will obey this type of relation approximately. The temperature dependence of γ' is unknown and in this case will be assumed to be temperature independent.

Equation (13b) may now be re-written thus

$$p_c = \alpha\sigma_{00}e^{-\chi T} - \left\{\frac{16G\gamma'}{\pi(1-\nu)k_0e^{-\epsilon T}} - \alpha k_0e^{-\epsilon T}\right\} d^{-\frac{1}{2}}.$$

Eliminating constants, this becomes

$$p_c = \xi e^{-\chi T} - \left(\zeta e^{\epsilon T} - \lambda e^{-\epsilon T}\right) d^{-\frac{1}{2}} \tag{17}$$

where $\xi = \alpha\sigma_{00}$, $\zeta = \dfrac{16G\gamma'}{\pi(1-\nu)k_0}$ and $\lambda = \alpha k_0$.

Expanding equation (17) and ignoring second order terms (χ and ϵ are both small), we get

$$p_c = \xi(1 - \chi T) - \{\zeta(1 + \epsilon T) - \lambda(1 - \epsilon T)\}d^{-\frac{1}{2}}$$

or

$$p_c = \xi - (\zeta - \lambda)d^{-\frac{1}{2}} - \xi\chi T - \epsilon(\zeta + \lambda)d^{-\frac{1}{2}}T. \tag{18}$$

For constant grain size, this reduces to Pugh's[3] equation (2) with

$$A' = \frac{\xi - (\zeta - \lambda)d^{-\frac{1}{2}}}{\xi\chi + \epsilon(\zeta + \lambda)d^{-\frac{1}{2}}}$$

$$B' = \frac{1}{\xi\chi + \epsilon(\zeta + \lambda)d^{-\frac{1}{2}}}$$

5. CONCLUSIONS

A theory for the brittle-ductile transition pressure has been developed and applied to the transition in zinc. It is shown that the quantities involved are of the right order. The theory predicts that the transition pressure increases both with σ_0 the frictional stress opposing the motion of a free dislocation (which may be increased by strain or quench-ageing and irradiation damage) and with increasing grain size.

The effect of temperature on the transition pressure is shown to be consistent with the empirical relationships of Pugh[1,3] and Galli and Gibbs[2].

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LIST OF FIGURES

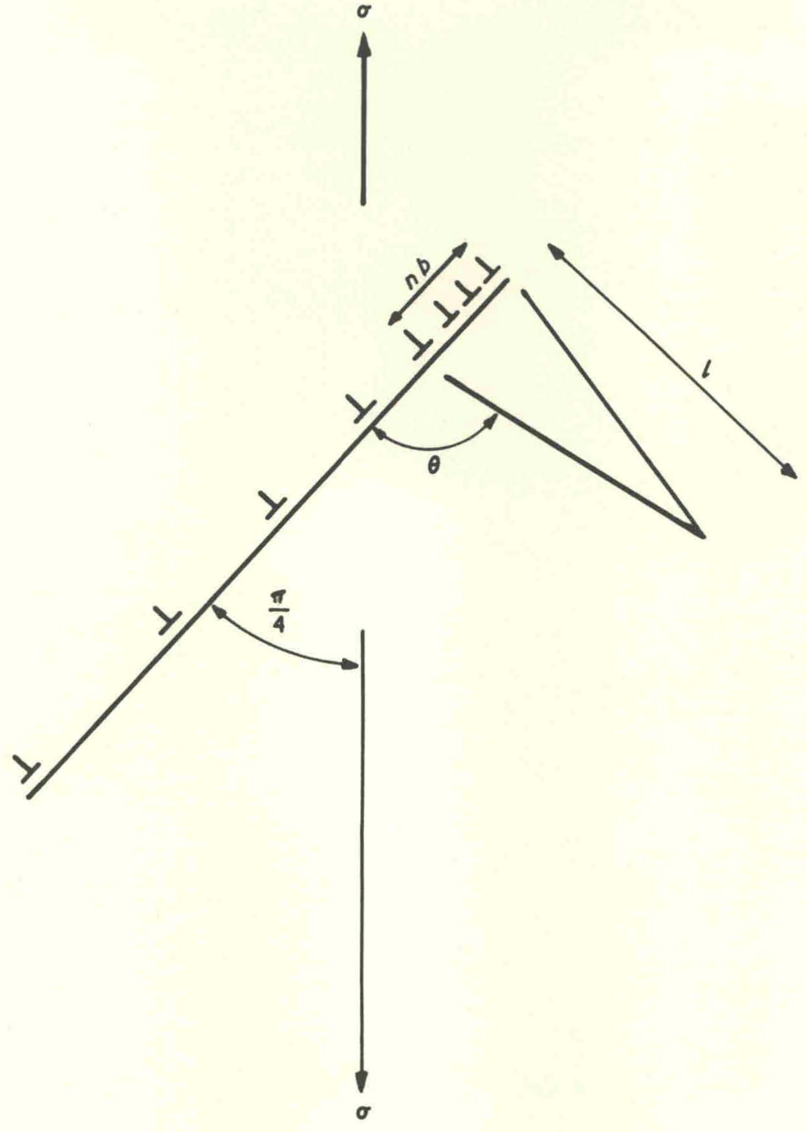

FIG I THE FORMATION OF A CRACK

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